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# DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

**181. Proposed by A. H. HOLMES, Brunswick, Maine.**

Sum the series, $1+2^m+3^m+4^m+\dots+n^m$.

**Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

An elementary solution by the theory of undetermined coefficients is as follows: Denoting the sum by $Sx^m$, we put

$$Sx^m=Ax^{m+1}+Bx^m+Cx^{m-1}+Dx^{m-2}+\dots$$

Since $(x+1)^m=S(x+1)^m-Sx^m$, we have

$$(x+1)^m=A[(x+1)^{m+1}-x^{m+1}]+B[(x+1)^m-x^m]+C[(x+1)^{m-1}-x^{m-1}]+\dots$$

Developing and arranging, we get

$$x^m+\binom{m}{1}x^{m-1}+\binom{m}{2}x^{m-2}+\binom{m}{3}x^{m-3}+\dots$$

$$=A\binom{m+1}{1}x^m+\left[A\binom{m+1}{2}+B\binom{m}{1}\right]x^{m-1}+$$

$$\left[A\binom{m+1}{3}+B\binom{m}{2}+C\binom{m-1}{1}\right]x^{m-2}+\dots+A+B+C+D+\dots$$

$$\therefore A\binom{m+1}{1}=1,\ A\binom{m+1}{2}+B\binom{m}{1}=\binom{m}{1};$$

$$A\binom{m+1}{3}+B\binom{m}{2}+C\binom{m-1}{1}=\binom{n}{2},\ \text{etc.}$$

$$\therefore A=\frac{1}{m+1},\ B=\tfrac{1}{2},\ C=\frac{m}{12},\ D=0,\ E=\frac{-m(m-1)(m-2)}{240},\ F=0,\ \text{etc.}$$

$$\therefore Sx^m=\frac{x^{m+1}}{m+1}+\tfrac{1}{2}x^m+\tfrac{1}{6}\cdot\tfrac{1}{2}\binom{m}{1}x^{m-1}-\tfrac{1}{30}\cdot\tfrac{1}{4}\binom{m}{3}x^{m-3}+\dots,$$ in which $\frac{1}{6}$, $\frac{1}{30}$,

etc., are Bernoulli's numbers.

Dr. Zerr referred to the solutions of this problem in Todhunter's Algebra, pp. 415-416, and Hall and Knight's Algebra, pp. 336-337.